Optimal Policy of Pitch-Hold Phase for Mine Detection of UAV based on Mixed-Integer Linear Programming

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Abstract. An optimization problem is formulated to maximize the coverage of the detection area for a tilt-rotor unmanned aerial vehicle (UAV), which performs a landmine detection mission. UAV has mine detection devices including magnetic field sensors and chemical sensors. Due to the characteristics of the detection sensors, UAV should fly at a certain altitude above the ground while following the terrain to perform accurate mine detection. Also, the UAV is required to maintain the pitch angle around the trim value when changing the altitude. Consequently, the altitude change rate of the UAV is limited during the mission. To maximize the mine detection capability within the limit, in this study, the entire flight interval is divided into the intervals on which the mines can be detected accurately and the intervals on which accurate detection is not possible. Logical variables are introduced to indicate the satisfaction of the requirements for the accurate detection. Constraints on the logical variables are transformed into a set of linear inequalities containing continuous variables and binary integer variables. Finally, the optimization problem is formulated as a mixed-integer linear programming problem. The effectiveness of the proposed method is demonstrated through numerical simulation.

Keywords: Mixed-integer linear programming (MILP), Unmanned aerial vehicle (UAV), Terrain-following, Mine detection

1 Introduction

The demilitarized zone (DMZ) in the Republic of Korea is a region where more than 1 million landmines are buried in its soil. Most of the remaining landmines were buried about 70 years ago. They have no utility for military purposes anymore. However, the landmines may pose a potential danger to civilians, because no one knows where and how many landmines are buried in DMZ. Therefore, a safe and efficient detection and removal of the landmines is necessary.

The usage of the UAV is widening for various military and civilian applications with the advances in unmanned aerial vehicle (UAV) technology. UAV has the advantage of being able to fly over large areas quickly and is less constrained by various forms of obstacles than humans or ground vehicles. To exploit the capabilities of UAVs for mine detection, certain R&D projects are in progress to develop a tilt-rotor UAV equipped with detecting devices including magnetic field sensors and chemical sensors [4], [13]. The mission of the UAV is to detect the mines in the ground and locate the exact location of the mines.

The UAV is required to fly at a certain altitude above the ground for the mine detection, which can be considered as a special case of aircraft terrain-following. Some research has been conducted regarding trajectory planning, guidance and control for terrain-following flight. In [8], the terrain-following was formulated as a trajectory optimization problem, where the objective is to maintain the trajectory close to the terrain and minimize the flight time. Point-mass nonlinear dynamics were considered, and an

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inverse dynamics approach was applied to solve the problem. In [10], a linear combination of Chebyshev polynomials was used to construct a terrain continuous mathematical model from a terrain discrete data, and a minimum time problem was solved based on the model. In [12], stair and optimal spline algorithms were proposed for the computation of the reference trajectory in the vertical plane. In [6], [7], terrain-following flight method was presented based on low-cost laser range measurement and preview control. A slightly different but closely related topic is a terrain collision avoidance. In [9], an efficient method to solve a general terrain avoidance problem in three-dimensional space was presented based on satellite elevation maps and quad-trees. In [5], terrain- and threat-based trajectory planning and control problems were addressed. A high-fidelity dynamic model was used, and the optimization problem was solved using evolutionary optimization algorithms.

Most of the research considered the terrain as an obstacle that should be avoided or passed by as quickly as possible. However, the terrain itself is of much interest for the mine detection as considered in this study. In the mine detection mission of the UAV, the objective of the terrain flight is to cover as much as portion of the area with high accuracy of detection as possible.

Two conditions should be met for accurate mine detection. First, the UAV should maintain a constant clearance from the ground because the magnitude of the detection signal is sensitive to the distance from the mine. Second, the UAV should regulate the pitch angle near its trim value when changing altitude or the roll angle when turning, because the detection sensors are mounted on the UAV without a gimbal. In this study, two-dimensional terrain on the vertical plane is considered, and only the requirements on the altitude and pitch angle are taken into account.

A tilt-rotor UAV considered in this study has additional control surfaces compared to fixed-wing aircraft, enabled to change altitude while keeping the variation of the pitch angle small. Although, the maximum achievable rate of altitude change of the UAV is limited when the pitch angle is held. One way to deal with this constraint is to solve the optimization problem that minimizes the cumulative altitude error over the whole flight interval with the constraint on the flight path angle. This approach provides acceptable performance for the terrain having a moderate slope. However, the performance of the mine detection may be degraded as the slope of the terrain gets steeper.

In this study, as an alternative, an optimization problem is formulated that maximizes the length of the flight interval on which the requirements for the altitude and pitch angle are satisfied. The entire flight interval is divided into phases named as *detection phase* and *non-detection phase*. Logical variables are introduced to indicate whether or not the requirements are satisfied. Then, the optimization problem is formulated as a mixed-integer linear programming (MILP) containing binary integer variables.

The paper is organized as follows. Section 2 reviews several useful properties of logical variables that are used in the paper. Section 3 presents the UAV model and the definition of the detection phase and non-detection phase. Then, the formulation for the MILP problem is described. Additional modifications to the optimization problem are also introduced. In Section 4, optimization result and numerical simulation result are presented to demonstrate the effectiveness of the proposed method. Finally, Section 5 summarizes the concluding remarks.

2 Preliminary

Logical variables are used in the optimization to indicate whether or not the conditions for accurate detection are satisfied. A framework for transforming the logical statements involving continuous variables into linear inequalities can be found in [1]. Some of the results are briefly reviewed and extended here.

A mathematical statement is expressed inside bracket, [], in this paper. A logical variable δ_i can be associated with the statement, which has a value of either 1 or 0, because a mathematical statement has a truth value of either true or false. $\delta_i = 1$ indicates that the statement is true, and $\delta_i = 0$ indicates that the statement is false.

Given two logical variables δ_1 and δ_2, their product $\delta = \delta_1\delta_2$ represents a compound statement combined with connectives $\wedge$ (and) because $\delta = 1$ if and only if both statements corresponding to δ_1 and δ_2 are satisfied. The relation can be equivalently expressed by the following linear inequality equations

$$\delta \leq \delta_1, \delta \leq \delta_2, \quad \delta \geq \delta_1 + \delta_2 - 1 \tag{1}$$

where each of $\delta, \delta_1, \delta_2$ is constrained to have value of either 1 or 0. In general, the relation $\delta = \prod_{i=1}^{n} \delta_i$ is equivalent to the following linear expression.

$$\delta \leq \delta_i \ \ (\forall i = 1, 2, \cdots, n), \quad \delta \geq 1 + \sum_{i=1}^{n} \delta_i - n \tag{2}$$

For $x \in \mathbb{R}^d, E \in \mathbb{R}^{1\times d}, f \in \mathbb{R}$, let us define a logical variable δ as

$$[Ex \leq f] \leftrightarrow [\delta = 1] \tag{3}$$

Assume that $x \in \mathcal{X}$ where $\mathcal{X}$ is a bounded set. Defining $M = \max_{x\in\mathcal{X}}(Ex), m = \min_{x\in\mathcal{X}}(Ex)$, the definition (3) is equivalent to

$$(f + \epsilon - m)\delta \geq f + \epsilon - Ex, \quad (M - f)(1 - \delta) \geq Ex - f \tag{4}$$

where ϵ is a small tolerance for which the machine precision is typically used.

For $x \in \mathbb{R}^d, E \in \mathbb{R}^{n\times d}, f \in \mathbb{R}^n$, let us define logical variables $\delta_i \ (i = 1, 2, \cdots, n)$ as

$$[(Ex)_i \leq f_i] \leftrightarrow [\delta_i = 1] \ \ (i = 1, 2, \cdots, n) \tag{5}$$

where $(Ex)_i$ and f_i represent the ith component of Ex and f, respectively. The statement $[Ex \leq f]$ is regarded as true if and only if $(Ex)_i \leq f_i$ is satisfied for all $i = 1, 2, \cdots, n$. Now, let us define a logical variable δ as

$$[Ex \leq f] \leftrightarrow [\delta = 1] \tag{6}$$

Then, the definition (6) is equivalent to

$$\delta \leq \ \delta_i \ \ (i = 1, 2, \cdots, n), \quad \delta \geq 1 + \ \sum_{i=1}^{n} \delta_i - n \tag{7}$$

where each δ_i is subject to the following constraint.

$$(f_i + \epsilon - m_i)\delta_i \geq \ f_i + \epsilon - (Ex)_i, \quad (M_i - f_i)(1 - \delta_i) \geq (Ex)_i - f_i \tag{8}$$

3 Problem Formulation

In this section, UAV model and the definition of the detection phase and non-detection phase are described. Logical variables are introduced to distinguish the two phases. A MILP problem is formulated to maximize the portion of the detection phase. The constraints are given in the form of linear equalities and inequalities. Some of the key components related to the problem are illustrated in Figure 1, which will be explained in the subsequent subsections. At the end of the section, a model predictive controller design is briefly presented for comparison with the proposed method.

3.1 UAV Model

The kinematic equations of the UAV model can be expressed as

$$\dot{p} = V\cos\gamma, \quad \dot{h} = V\sin\gamma \tag{9}$$

where V is the airspeed, γ is the vertical flight path angle, h is the altitude, and p is the downrange. It is assumed that the UAV maintains constant airspeed. Time response of the actual flight path angle γ with

respect to the flight path angle command γ_c is approximated as a first-order linear model to incorporate longitudinal dynamics of the vehicle as

$$\frac{\gamma(s)}{\gamma_c(s)} = \frac{1}{\tau s+1} \tag{10}$$

where τ denotes the time constant of the model.

The maximum achievable rate of altitude change (or equivalently the magnitude of the flight path angle) is limited when the UAV changes the altitude with pitch angle held (pitch-hold case). The achievable magnitude of the flight path angle is larger than the pitch-hold case when the UAV changes the altitude without pitch angle held (pitch-free case). Accordingly, the range of the achievable flight path angle is expressed as

$$\gamma_{min,ph} \leq \gamma \leq \gamma_{max,ph}, \quad \gamma_{min,free} \leq \gamma \leq \gamma_{max,free} \tag{11}$$

for the pitch-hold case and pitch-free case, respectively.

3.2 Detection Phase and Non-Detection Phase

In this study, it is assumed that magnetic field sensors and chemical sensors are used for the mine detection. The UAV should fly maintaining a constant clearance h_{off} above the ground for accurate detection due to the characteristics of the detection sensors. Furthermore, the maximum achievable magnitude of the flight path angle of the UAV is limited due to the pitch-hold maneuver. These two conditions can be expressed as

$$h_{ref} - h_{thr} \leq h \leq h_{ref} + h_{thr}, \quad \gamma_{min,ph} \leq \gamma \leq \gamma_{max,ph} \tag{12}$$

where h_{ref} is the reference altitude for the UAV to achieve the terrain-following flight, and h_{thr} is the allowable altitude error. The reference altitude h_{ref} is determined as $h_{ref} = h_{ter} + h_{off}$ for the given terrain elevation h_{ter} and the desired clearance h_{off}. The UAV is said to be in the *detection phase* when both conditions are satisfied. On the other hand, the UAV is said to be in the *non-detection phase* when any of the conditions is not satisfied. In this study, time intervals of the *detection phase* are defined as *detecting intervals*, and time intervals of the *non-detection phase* are defined as *passing intervals*.

The length of *detecting intervals* should be maximized for the UAV to explore large areas quickly and identify the exact location of the mines. Physically, the UAV is not able to follow the terrain with pitch angle held for the flight intervals where the slope of the terrain is steeper than the maximum achievable flight path angle of the UAV constrained by the pitch-hold maneuver. This poses a problem because mine detection becomes inaccurate due to the altitude error if the UAV sticks to pitch-hold maneuver, while mine detection becomes inaccurate due to the change in the pitch angle if the UAV gives up pitch-hold maneuver. In other words, both conditions on altitude and pitch angle cannot be simultaneously satisfied during these intervals.

However, if the pitch-hold constraint is not imposed on the UAV, the UAV can change the altitude more quickly, which allows more flexibility to the entire mission. A better solution may be obtained if the UAV gives up detecting the mines in some portion of the flight interval where the slope is steep and instead concentrates on the other portion of the flight interval. A decision on which portion to be chosen as *detecting intervals* and which portion to be chosen as *passing intervals* is called as a policy in this study.

To find the optimal policy maximizing the length of *detecting intervals*, an optimization problem is formulated, which will be discussed in the following subsection.

3.3. Mixed-Integer Linear Programming (MILP)

First, the essential part of the optimization problem is presented including dynamic constraints, constraints on the *detection phase*, and the objective function. Then, two additional aspects of the problem are presented.

Basic Formulation

Consider that a terrain profile is given where the UAV is supposed to search for the mines. The initial downrange beginning the search is denoted as p_i, and the final downrange ending the search is denoted as p_f. Given p_i and p_f, the downrange interval $[p_i, p_f]$ is discretized into N subintervals $[p_0, p_1]$, $[p_1, p_2]$, ⋯, $[p_{N-1}, p_N]$, where each subinterval has the same length. It is assumed that the UAV maintains the constant airspeed V and flies at a small flight path angle γ. Under these assumptions, the time taken to travel each subinterval becomes approximately same. Consequently, the total time interval corresponding to the downrange interval can also be discretized into N time intervals having the same length. The discretized time intervals are denoted as $[t_0, t_1]$, $[t_1, t_2]$, ⋯, $[t_{N-1}, t_N]$, where $t_{k+1} - t_k = T_d$ $(k = 0, 1, \cdots, N-1)$, and the discretization time interval T_d is given as

$$T_d = \frac{p_f - p_i}{NV} \tag{13}$$

Each endpoint of the downrange intervals is associated with the corresponding time instant t_k as shown in Figure 1.

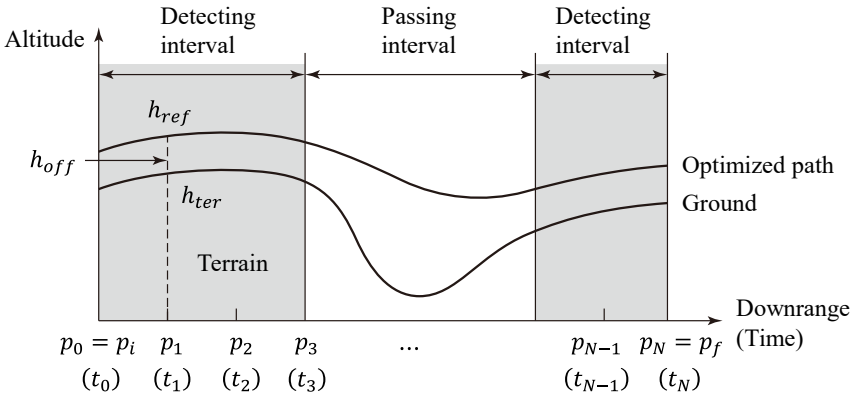


Fig. 1. Terrain-following flight with distinction between detecting intervals and passing intervals

The UAV model given in Subsection 3.1 is linearized and discretized. Assuming that the flight path angle of the UAV is small, the UAV model can be linearized as follows:

$$\begin{bmatrix} \dot{h} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} 0 & V \\ 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} h \\ \gamma \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\tau} \end{bmatrix} \gamma_c = Ax + Bu \tag{14}$$

where $x = [h \ \ \gamma]^T$ is used as the state and $u = \gamma_c$ is used as the control input for the UAV. Note that the equation for $\dot{p}$ has been dropped, because the change for the downrange is approximately constant with respect to time. The continuous model, Eq. (14), can be discretized with time interval T_d as

$$x(k+1) = A_d x(k) + B_d u(k), \quad A_d = e^{AT_d}, \quad B_d = \left(\int_0^{T_d} e^{A\xi} d\xi \right) B \tag{15}$$

where the state and control input of the UAV at time instant t_k are denoted as $x(k)$ and $u(k)$, respectively.

The condition for the *detection phase* at each time instant t_k can be expressed as

$$Ex(k) \le f(k), \quad E = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}, \quad f(k) = \begin{bmatrix} \overline{h}(k) \\ -\underline{h}(k) \\ \overline{\gamma} \\ -\underline{\gamma} \end{bmatrix} \tag{16}$$

where the maximum allowable altitude $\overline{h}(k)$ and the minimum allowable altitude $\underline{h}(k)$ are defined as

$$\overline{h}(k) = h_{ref}(k) + h_{thr}, \quad \underline{h}(k) = h_{ref}(k) - h_{thr} \tag{17}$$

with the reference altitude $h_{ref}(k)$ at t_k. For the maximum allowable flight path angle $\overline{\gamma}$ and the minimum allowable flight path angle $\underline{\gamma}$, the same values are used for the entire flight interval as

$$\overline{\gamma} = \gamma_{max,ph}, \underline{\gamma} = \gamma_{min,ph} \tag{18}$$

To indicate the satisfaction of conditions for the *detection phase*, logical variables $\delta(k)$ and $\delta_1(k)$, $\delta_2(k)$, $\delta_3(k)$, $\delta_4(k)$ are defined as follows:

$$[Ex(k) \le f(k)] \leftrightarrow [\delta(k) = 1] \tag{19}$$

$$\left[\left(Ex(k)\right)_i \le f_i(k)\right] \leftrightarrow [\delta_i(k) = 1] \ \ (i = 1, 2, 3, 4) \tag{20}$$

where $\delta(k)$ indicates whether the inequality $Ex(k) \le f(k)$ is satisfied or not. Each $\delta_i(k)$ $(i = 1, 2, 3, 4)$ indicates whether the i-th row of the inequality is satisfied or not. The inequality in Eq. (19) is applied as an element-wise inequality to each row. That is,

$$[\delta(k) = 1] \leftrightarrow [\delta_i(k) = 1] \ (\forall i) \tag{21}$$

Let us define the following objective function,

$$J = \sum_{k=0}^{N}\left(1 - \delta(k)\right) + w_1 \sum_{k=0}^{N-1}|u(k)| + w_2 \sum_{k=1}^{N-1}|u(k) - u(k-1)| \tag{22}$$

where w_1 and w_2 are weights. The objective function consists of three terms. The first term represents the total number of points where the conditions for the *detection phase* is *not* satisfied. In other words, the first term represents the total number of points in the *non-detection phase*. Therefore, minimizing J results in *minimizing* the portion of the *non-detection phase* (*maximizing* the portion of the *detection phase*). The second and third terms are included to obtain the trajectory with less control effort and more smooth change in the control input.

To find the optimal policy, the objective function J is minimized,

$$\min_{u(k),\delta(k),\delta_i(k)} J \tag{23}$$

subject to Eqs. (15), (19), and (20) as constraints. For Eqs. (19) and (20), the constraints are transformed into linear inequalities as in Eqs. (7) and (8). Note that logical variables $\delta(k)$ and $\delta_i(k)$ are binary variables, which can be seen as special cases of integer variables. In addition, the objective function can be transformed into a linear objective function with additional linear inequalities using slack variables. Therefore, the above optimization problem is one of the MILP problems, which can be solved efficiently using various MILP solvers.

State Constraints to Ensure Safety

Ensuring the collision avoidance from the ground is important because the UAV flies above the ground with a relatively small amount of clearance. A safety constraint to ensure collision avoidance can be easily expressed in a linear inequality. Given a safety distance h_{safe} that should be at least cleared from

the ground, the minimum safe altitude for the UAV to avoid collision can be set as $h_{ref}(k) - h_{off} + h_{safe}$. Thus, the safety constraint at each time instant t_k can be expressed as

$$[1\ \ 0]x(k) \geq h_{ref}(k) - h_{off} + h_{safe} \tag{24}$$

where the constraint should be applied both on the *detecting intervals* and *passing intervals*.

Modification for Effective Detection

Note that it is not enough to consider the conditions for the detection phase only at a single distinct time instant as in Eq. (21), because the optimization may lead to an undesirable solution with chattering control inputs if the simple constraint in Eq. (21) is used. This is due to the discrete nature of the formulation. The conditions are modified to be satisfied not only at t_k but also at the adjacent time instants t_{k-1} and t_{k+1} to deal with the issue. That is, the constraint in Eq. (21) is modified as follows:

$$[\delta(k) = 1] \leftrightarrow [\delta_i(k-1) = 1] \wedge [\delta_i(k) = 1] \wedge [\delta_i(k+1) = 1]\ (\forall \mathrm{i}) \tag{25}$$

where ∧ denotes 'and' operator.

3. 4. Model Predictive Control (MPC)

A model predictive controller [4] is adopted for the altitude tracking controller. The controller does not distinguish intervals for the detection phase and non-detection phase. That is, the pitch angle is assumed to be held for the entire flight.

Let us define $e(k) = h(k) - h_{ref}(k)$ as the altitude error at instant t_k. A linear MPC problem can be formulated as finding the control sequence $U = \{u(0), \cdots, u(N_p - 1)\}$ that minimizes

$$J = e(N_p)^T P e(N_p) + \sum_{k=0}^{N_p-1} e(k)^T Q e(k) + \sum_{k=0}^{N_p-1} u(k)^T R u(k) \tag{26}$$

subject to the dynamic constraint in Eq. (15) and the control constraint

$$\gamma_{min,ph} \leq u(k) \leq \gamma_{max,ph}\ (k = 0, 1, \cdots, N_p - 1) \tag{27}$$

where N_p denotes the prediction horizon length, and P, Q, R denote weights. The same discretized model is used for the controller as in the MILP problem. The control sequence is optimized at every control input update time by taking the current state as the initial state, because MPC acts in a receding horizon manner. In this study, the update period T_c for the control input is set as a much smaller value than the discretization time interval T_d [4].

4 Numerical Simulation

Numerical simulation is performed to demonstrate the effectiveness of the proposed method. The terrain model used in the numerical simulation is constructed from digital elevation model (DEM) data provided on National Spatial Data Infrastructure Portal [11]. The DEM data is provided in grid space form where the coordinates of the horizontal position and terrain elevation are associated for each point spaced at regular intervals along the North and East direction. A horizontal path for the UAV is set on the terrain after the mission area is chosen. A two-dimensional terrain profile is obtained by querying the terrain elevation at the horizontal position corresponding to each point along the path. The terrain elevation at a point other than on the grid is approximated using bilinear interpolation from near grid points.

The DEM data for Yeoncheon-Gun, Gyeonggi-do, the Republic of Korea, is chosen as a terrain model for the numerical simulation. A straight horizontal path of 2.5 km length is set. Figure 2 illustrates the terrain profile and the horizontal path. The maximum and minimum elevation of the terrain along the horizontal path is 179.0 m and 104.3 m, respectively. The maximum absolute slope is 15.0 deg, and the

average absolute slope is 3.4 deg. The clearance h_{off} between the UAV and the ground is set as 30 m. The initial altitude $h(0)$ of the UAV is set as 30 m above the ground, which means that the UAV begins the flight at the desired altitude. The initial flight path angle $\gamma(0)$ of the UAV is set as zero.

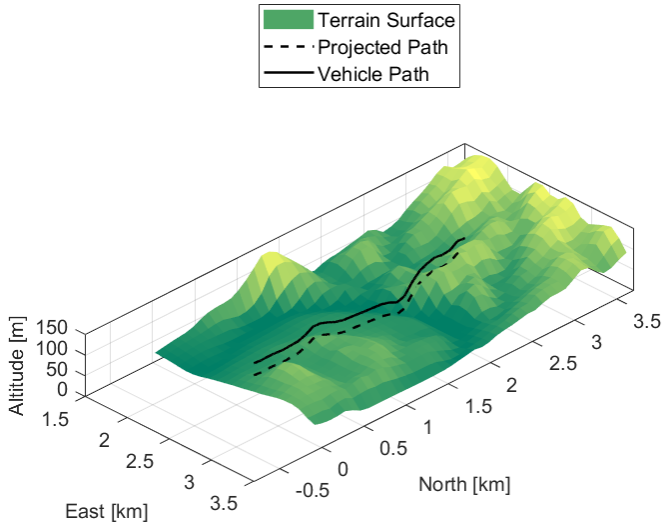


Fig. 2 An illustration of the terrain profile and the horizontal path

The optimization problem described in Section 3 is solved using CVX [2] and Gurobi [3]. Safety constraints as in Eq. (24) are included, and modified constraints on logical variables as in Eq. (25) are used instead of the original constraints. The parameters for the UAV model and optimization problem are summarized in Tables 1 and 2.

Table 1 UAV model parameters

Parameters	V	τ	$\gamma_{max,ph}, \gamma_{min,ph}$	$\gamma_{max,free}, \gamma_{min,free}$	h_{off}
Values	25 m/s	0.5 s^{-1}	$2.5, -2.5$ deg	$10, -10$ deg	30 m

Table 2 Optimization problem parameters

Parameters	w_1, w_2	N	T_d	h_{safe}	h_{thr}
Values	10, 1	100	1 s	10 m	1 m

For the model predictive controller, $N_p = 20$ is used for the prediction horizon length, $T_d = 1$ for the discretization time, $T_c = 0.05$ for the control update interval, and $P = 1, Q = 1, R = 32$ for the weights.

Figure 3 shows the optimal policy obtained by solving the MILP problem. The shaded regions in each figure indicate *detecting intervals*. Other regions without shaded indicate *passing intervals*. The top figure shows the terrain elevation and the optimal flight trajectory. As shown in the figure, intervals with the moderate terrain slope are chosen as *detecting intervals,* and intervals with the steep terrain slope are chosen as *passing intervals*, as expected. The middle figure shows the optimal control trajectory. The bottom figure shows the altitude error for the reference altitude. The red dotted lines in the middle figure represent the achievable limit of the flight path angle with the pitch-hold maneuver. The red dotted lines in the bottom figure represent the allowable limit of the altitude error for accurate detection. In the second *passing interval*, the UAV climbs at a much higher rate as the pitch-hold maneuver was given up. As a result, the UAV can secure the *detecting interval* on the latter part of the terrain.

Figure 4 shows the numerical simulation result when the MPC is used instead of the MILP. As mentioned in the previous section, the MPC does not distinguish intervals regardless of whether the slope of the terrain is steep or not. The UAV just tries its best to follow the terrain blindly. As a result, the UAV fails to perform detection for most of the latter part.

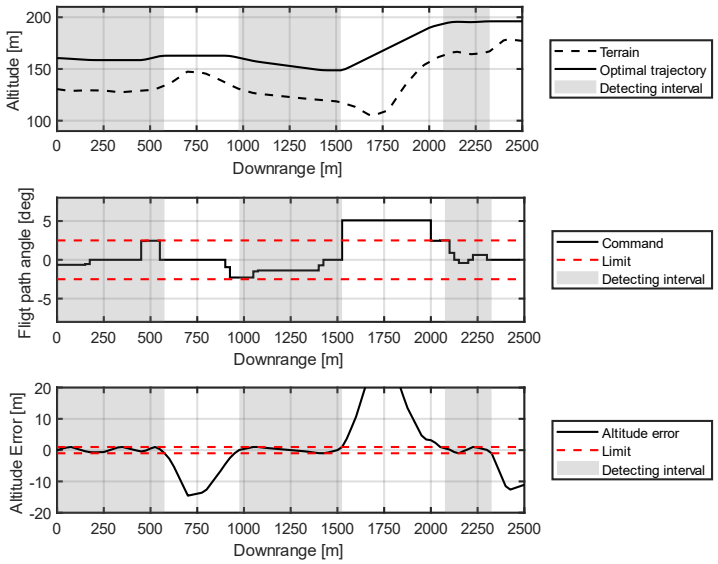


Fig. 3 The optimal policy obtained by solving the MILP problem

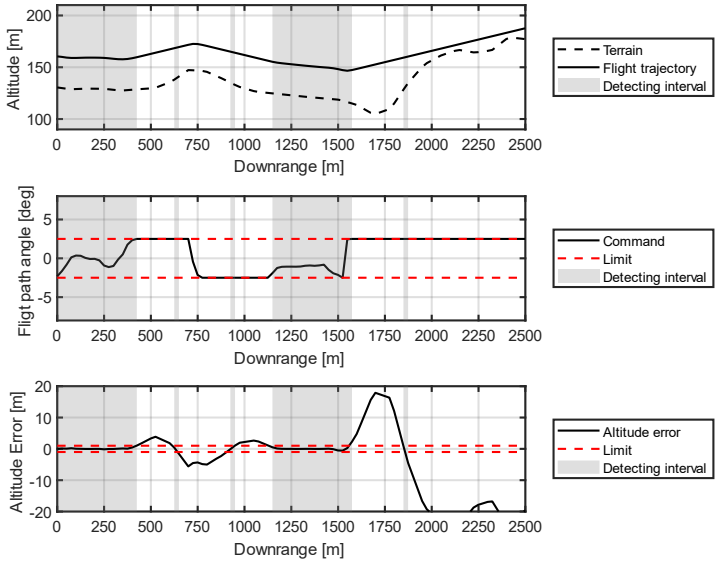


Fig. 4 The numerical simulation result obtained using the MPC.

The portion of the *detecting intervals* in the entire flight is 55.0% for the MILP case and 37.0% for the MPC case, respectively. Note that, in the MILP case, the UAV passes by the first *passing interval* almost straight, because there is no need to follow the terrain while maintaining the exact clearance h_{off} on the interval. From this, it can be inferred that the proposed method also has an advantage in the aspect of total control effort.

5 Conclusion

An optimization problem was formulated as a MILP problem to maximize the length of *detecting intervals* for the UAV searching for the landmines. The requirements for the *detecting intervals* were

defined and logical variables were introduced to indicate the satisfaction of the requirements. The effectiveness of the proposed method was shown by comparing the optimization result with the result using MPC. It can be stated that the optimal policy properly decides on which interval to follow and pass by the terrain.

The formulation in this paper can be thought of as constructing the high-level policy for the mine detection mission. A unified structure is necessary for accurate detection that can achieve the minimum altitude error in *detecting intervals* determined by the policy. Our future work includes integrating the proposed method with controllers at the lower level. Extending the proposed method to a three-dimensional terrain-following case will also be an interesting problem.

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References

1. Bemporad A, Morari M (1999) Control of systems integrating logic, dynamics, and constraints. Automatica. 35(3):407-27
2. Grant M, Boyd S (2014) CVX: Matlab software for disciplined convex programming, version 2.1. Available via https://cvxr.com/cvx/. Cited 26 Aug 2021
3. Gurobi Optimization, LLC (2021) Gurobi Optimizer reference manual. Available via https://www.gurobi.com. Cited 26 Aug 2021
4. Kang H, Lee S, Lee J, Kim Y, Suk J, Kim S (2021) Altitude Tracking of UAV with Pitch-Hold Constraint Based on Model Predictive Control for Mine Detection. The 21st International Conference on Control, Automation and Systems (ICCAS 2021) Jeju, Korea
5. Kamyar R, Taheri E (2014) Aircraft optimal terrain/threat-based trajectory planning and control. Journal of Guidance, Control, and Dynamics. 37(2):466-83
6. Livshitz A, Idan M (2018) Low-cost laser range-measurement-based terrain-following concept and error analysis. Journal of Guidance, Control, and Dynamics. 41(4):1006-14
7. Livshitz A, Idan M (2019) Preview Control approach for laser-range-finder-based terrain following. IEEE Transactions on Aerospace and Electronic Systems. 56(2):1318-31
8. Lu P, Pierson BL (1995) Optimal aircraft terrain-following analysis and trajectory generation. Journal of guidance, Control, and Dynamics. 18(3):555-60
9. Malaek S, Abbasi A (2011) Near-optimal terrain collision avoidance trajectories using elevation maps. IEEE Transactions on Aerospace and Electronic Systems.47(4):2490-501
10. Malaek S, Kosari A (2007) Novel minimum time trajectory planning in terrain following flights. IEEE Transactions on Aerospace and Electronic Systems. 43(1):2-12
11. Ministry of Land, Infrastructure and Transport (2021) National spatial data infrastructure portal. Available via http://www.nsdi.go.kr. Cited 26 Aug, 2021
12. Samar R, Rehman A (2011) Autonomous terrain-following for unmanned air vehicles. Mechatronics. 21(5):844-60
13. Yoo L, Lee J, Lee Y, Jung S, Choi Y (2021) Application of a drone magnetometer system to military mine detection in the demilitarized zone. Sensors. 21(9):3175

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